נוסחאות אינטגרלים

1 אינגרלים אלמנטריים ונוסחאות יסוד

$$\int a\,dx = ax \tag{1}$$

$$\int a \cdot f(x)\,dx = a\int f(x)\,dx \tag{2}$$

$$\int \phi(y)\,dx = \int \frac{\phi(y)}{y'}\,dy \qquad y' = \frac{dy}{dx} \tag{3}$$

$$\int (u+v)\;dx = \int u\,dx + \int v\,dx \tag{4}$$

$$\int u\,dv = uv - \int v\,du \tag{5}$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx \tag{6}$$

$$\int f(ax)\,dx = \frac{1}{a}\int f(u)\,du \tag{7}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq 1 \tag{8}$$

$$\int \frac{f'(x)\,dx}{f(x)} = \ln f(x) \qquad f'(x) = \frac{df(x)}{dx} \tag{9}$$

$$\int \frac{dx}{x} = \ln x \quad \text{, or } \ln(-x) \tag{10}$$

$$\int \frac{fi(x)\,dx}{2\sqrt{f(x)}} = \sqrt{f(x)} \qquad f'(x) = \frac{df(x)}{dx} \tag{11}$$

$$\int \mathrm{e}^x\,dx = \mathrm{e}^x \tag{12}$$

$$\int \mathrm{e}^{ax}\,dx = \frac{e^{ax}}{a} \tag{13}$$

$$\int b^{ax}\,dx = \frac{b^{ax}}{a\ln b} \tag{14}$$

$$\int \ln x\,dx = x\ln x - x \tag{15}$$

$$\int a^x \ln a\,dx = a^x \tag{16}$$

$$\int \sin x\,dx = -\cos x \tag{17}$$

$$\int \cos x\,dx = \sin x \tag{18}$$

$$\int \frac{dx}{\cos^2 x} = \tan x \tag{19}$$

$$\int \frac{dx}{\sin^2 x} = -\cot x \tag{20}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \tag{21}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x \tag{22}$$

$$\int \tan x \, dx = -\ln(\cos x) \tag{23}$$

$$\int \cot x \, dx = \ln(\sin x) \tag{24}$$

$$\int \frac{dx}{\sin x} = \ln\left(\tan\frac{x}{2}\right) \tag{25}$$

$$\int \frac{dx}{\cos x} = \ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right) \tag{26}$$

$$\int \sin^{-1} x \, dx = x\sin^{-1} x + \sqrt{1-x^2} \tag{27}$$

$$\int \tan^{-1} x \, dx = x\tan^{-1} x - \frac{1}{2}\ln(x^2+1) \tag{28}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) \tag{29}$$

$$\int \frac{dx}{\sqrt{x^2+b}} = \ln\left(x+\sqrt{x^2+b}\right) \qquad b \neq 0 \tag{30}$$

$$\int \sqrt{x^2+b} \, dx = \frac{1}{2}x\sqrt{x^2+b} + \frac{b}{2}\ln\left(x+\sqrt{x^2+b}\right) \qquad b \neq 0 \tag{31}$$

$$\int \sqrt{a^2-x^2} \, dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} \tag{32}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a}\sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{34}$$

2 אינטגרלים המכילים ביטויים מהצורה $ax+b$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b) \tag{35}$$

$$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)} \tag{36}$$

$$\int \frac{dx}{(ax+b)^3} = \frac{-1}{2a(ax+b)^2} \tag{37}$$

$$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b) \tag{38}$$

$$\int \frac{x^2\,dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b) \tag{39}$$

$$\int \frac{x^3\,dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b) \tag{40}$$

$$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\log\left(\frac{x}{ax+b}\right) \tag{41}$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right) \tag{42}$$

$$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right) \tag{43}$$

$$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b) \tag{44}$$

$$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3}(ax+b) - \frac{2b}{a^3}\ln(ax+b) \tag{45}$$

$$\int \frac{x^3\,dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{4b^2}{a^4}\ln(ax+b) \tag{46}$$

$$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right) + \frac{1}{b(ax+b)} \tag{47}$$

$$\int \frac{dx}{x^2(ax+b)^2} = \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right) - \frac{a}{b^2(ax+b)} - \frac{1}{b^2x} \tag{48}$$

$$\int \frac{dx}{x^3(ax+b)^2} = -\frac{3a^2}{b^4}\ln\left(\frac{ax+b}{x}\right) - \frac{(ax+b)^2}{2b^4x^2} + \frac{3a(ax+b)}{b^4x} - \frac{a^3x}{b^4(ax+b)} \tag{49}$$

$$\int \frac{x\,dx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^{2(}(ax+b)^2} \tag{50}$$

$$\int \frac{x^2\,dx}{(ax+b)^3} = \frac{\ln(ax+b)}{a^3} + \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} \tag{51}$$

$$\int \frac{x^3\,dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)^2} - \frac{3b}{a^4}\ln(ax+b) \tag{52}$$

$$\int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3}\ln\left(\frac{ax+b}{x}\right) \tag{53}$$

$$\int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4}\ln\left(\frac{ax+b}{x}\right) \tag{54}$$

$$\int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^2}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5}\ln\left(\frac{ax+b}{x}\right) \tag{55}$$

$$\int \frac{dx}{(ax+b)^n} = \frac{(ax+b)^{1-n}}{a(1-n)} \qquad n \neq 1 \tag{56}$$

$$\int \frac{x\,dx}{(ax+b)^n} = -\frac{(ax+b)^{1-n}(anx-ax+b)}{a^2(n-2)(n-1)} \qquad n \neq 1,2 \tag{57}$$

$$\int \frac{x^2\,dx}{(ax+b)^n} = \frac{1}{a^3}\left(\frac{-1}{(n-3)(ax+b)^{n-3}} + \frac{2b}{(n-2)(ax+b)^{n-2}} - \frac{a^2}{(n-1)(ax+b)^{n-1}}\right) \qquad n \neq 1,2,3 \tag{58}$$

$$\int (ax+b)^n\,dx = \tag{59}$$

$$\int x^2(ax+b)^n\,dx = \tag{60}$$

$$\int x^2(ax+b)^n\,dx = (n+1a \tag{61}$$

$$\int x^m(ax+b)^n\,dx = \tag{62}$$

$$\int x^m(ax+b)^n\,dx = \tag{63}$$

3 אינטגלים המכילים ביטויים מהצורה $\sqrt{ax+b}$

$$\int \frac{dx}{\sqrt{ax+b}} = \frac{2(ax+b)}{a} \tag{64}$$

$$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b} \tag{65}$$

$$\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(4a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b} \tag{66}$$

$$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}}\ln\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}} \end{cases} \tag{67}$$

$$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} \tag{68}$$

$$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{ax+b^3}}{3a} \tag{69}$$

$$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3} \tag{70}$$

$$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3} \tag{71}$$

$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}} \tag{72}$$

$$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}} \tag{73}$$

$$\int \frac{x^m}{\sqrt{ax+b}}\,dx = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+a)a}\int \frac{x^{m-1}}{\sqrt{ax+b}}\,dx \tag{74}$$

$$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int \frac{dx}{x^{m-1}\sqrt{ax+b}} \tag{75}$$

$$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}(ax+b)^{3/2} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx \tag{76}$$

$$\int \frac{\sqrt{ax+b}}{x^m} = \frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}} \tag{77}$$

$$\int \frac{\sqrt{ax+b}}{x^m} = \frac{-(ax+b)^{3/2}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int \frac{\sqrt{ax+b}}{x^{m-1}}\,dx \tag{78}$$

$$\int (ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+2)/2}}{a(m+2)} \tag{79}$$

$$\int x(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(ax+b)^{(m+2)/2}}{a^2(m+2)} \tag{80}$$

$$\int x^2(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+6)/2}}{a^2(m+6)} - \frac{4b(ax+b)^{(m+4)/2}}{a^3(m+4)} + \frac{2b^2(ax+b)^{(m+2)/2}}{a^3(m+2)} \tag{81}$$

$$\int \frac{(ax+b)^{m/2}}{x}\,dx = \frac{2(ax+b)^{m/2}}{m} + b\int \frac{(ax+b)^{(m-2)/2}}{x}\,dx \tag{82}$$

$$\int \frac{(ax+b)^{m/2}}{x^2}\,dx = -\frac{(ax+b)^{(m+2)/2}}{bx} + \frac{ma}{2b}\int \frac{(ax+b)^{m/2}}{x}\,dx \tag{83}$$

$$\int \frac{dx}{x(ax+b)^{m/2}} = \frac{2}{b(m-2)(ax+b)^{(m-2)/2}} + \frac{1}{b}\int \frac{dx}{x(ax+b)^{(m-2)/2}} \tag{84}$$